Section 7.3: The Class NP

These exercises reflect material from our text, *Introduction to the Theory of Computation*, by Michael Sipser, PWS Publishing Co., 1997, as well as *Introduction to Algorithms*, by Cormen, Leiserson, and Rivest, MIT Press, 1990.

Basic Concepts

Define the following concepts:

(a) P

(b) NP

The Class NP

(CLR 36.2-1) Consider the language

$$GRAPH-ISOMORPHISM = \{\langle G_1, G_2 \rangle : G_1 \text{ and } G_2 \text{ are isomorphic graphs}\}.$$

Prove that $GRAPH-ISOMORPHISM \in NP$ by describing a polynomial-time algorithm to verify the language.

(CLR 36.1-4) Show that the class NP of languages is closed under union, intersection, concatenation, and Kleene star. Discuss the closure of NP under complement.

(CLR 36.1-5) Show that any language in NP can be decided by an algorithm running in time $2^{\mathcal{O}(n^k)}$ for some constant k.

(CLR 36.1-8) Let ϕ be a boolean formula constructed from the boolean input variables $x_1, x_2, \ldots, x_k$, negations ($\neg$), AND'S ($\wedge$), OR'S ($\vee$), and parentheses. The formula ϕ is a *tautology* if it evaluates to 1 for every assignment of 1 and 0 to the input variables. Define $TAUTOLOGY$ as the language of boolean formulas that are tautologies. Show that $TAUTOLOGY \in coNP$.

(CLR 36.1-9) Prove that $P \subseteq coNP$.

(CLR 36.1-10) Prove that if $NP \neq coNP$, then $P \neq NP$.