

Section 7.3: Cliques

These exercises reflect material from our text, *Introduction to the Theory of Computation*, by Michael Sipser, PWS Publishing Co., 1997.

Basic Concepts

Define each of the following concepts:

(a) P

(b) NP

(c) Undirected graph, $G = (V, E)$

Describe two distinct ways of encoding undirected graphs so that they can be processed by computers.

Triangles

Let

$$TRIANGLE = \{\langle G \rangle : G \text{ is an undirected graph containing a triangle}\}$$

Construct a Turing machine which decides $TRIANGLE$.

Show that $TRIANGLE \in P$.

Hint: Calculate the runtime of the Turing machine just constructed.

On input $\langle G \rangle$ you should find a time complexity containing a factor of the form $\mathcal{O}\left(\binom{3}{2} \binom{|V|}{3}\right)$.

k-Cliques

For each positive integer k , let

$$CLIQUE(k) = \{\langle G, k \rangle : G \text{ is an undirected graph containing a } k\text{-clique}\}$$

Construct a Turing machine which decides $CLIQUE(k)$.

Show that $CLIQUE(k) \in P$.

Hint: Calculate the runtime of the Turing machine just constructed.

On input $\langle G, k \rangle$ you should find a time complexity containing a factor of the form $\mathcal{O}\left(\binom{k}{2} \binom{|V|}{k}\right)$.

Cliques

Let

$$CLIQUE = \{\langle G, k \rangle : G \text{ is an undirected graph containing a } k\text{-clique}\}$$

Construct a Turing machine which decides $CLIQUE$.

Show that $CLIQUE \in NP$.

Hint: Calculate the runtime of the Turing machine just constructed.

On input $\langle G, k \rangle$, you should find a time complexity containing a factor of the form $\mathcal{O}\left(\binom{k}{2} \binom{|V|}{k}\right)$. If k is fixed, this gives polynomial time, but if k is allowed to grow with n , then the runtime becomes exponential. Show, for instance, that $\binom{n/2}{2} \binom{n}{n/2}$ is larger than 2^n . Stirling's Formula might be helpful here.