Section 3.1: Derivatives of Polynomials and Exponential Functions

These notes reflect material from our text, *Calculus, Concepts and Contexts, Third Edition*, by James Stewart, published by Brooks/Cole, Pacific Grove, CA, 2005.

Key points from Stewart, Section 3.1: Deriving formulas for the derivative function.

The Derivative

Recall that if $f(x)$ is a function, the *derivative at* $x = a$ is given by

$$f'(a) = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$$

provided that this limit exists.

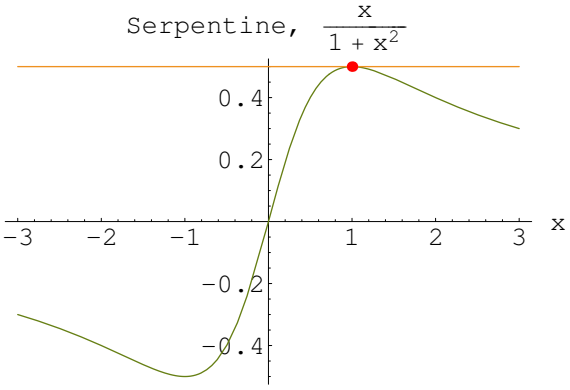

Fig. Derivatives are slopes of tangent lines.

Rules for Calculating the Derivative Function

Let $f'(x)$ and $g'(x)$ denote the derivative functions of $f(x)$ and $g(x)$ respectively and suppose that c, b, n and m are constants.

Constant Functions

$$\frac{d}{dx}(c) = 0$$

Linear Functions

$$\frac{d}{dx}(mx + b) = m$$

Constant Multiples

$$\frac{d}{dx}\big(c \cdot f(x)\big) = c \cdot f'(x)$$

Power Functions

$$\frac{d}{dx}(x^n) = n \cdot x^{n-1}$$

Sums and Differences

$$\frac{d}{dx}\big(f(x) \pm g(x)\big) = f'(x) \pm g'(x)$$

Summary of the Rules for Derivatives (so far)

$(cf)' = cf'$

$(f \pm g)' = f' \pm g'$

Rules to Calculate the Derivatives of Exponential Functions

Let a be a positive constant.

General Exponential Functions

$$\frac{d}{dx}(a^x) = (\ln a)a^x$$

Natural Exponential Function

$$\frac{d}{dx}(e^x) = e^x$$

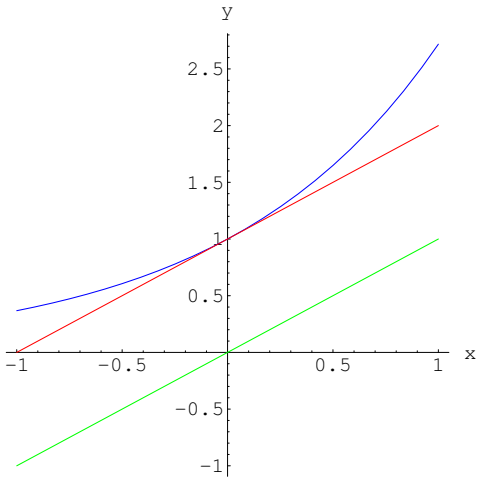

Fig. The graphs of $e^x, x + 1,$ and x.

Some Specific Derivatives

If $f(x) = c$, where c is a constant, then $f'(x) = 0$.

If $f(x) = x$, so that f is the identity function, then $f'(x) = 1$.

If $f(x) = x^n$, then $f'(x) = nx^{n-1}$, where n is a natural (counting) number.

If $f(x) = x^r$, then $f'(x) = rx^{r-1}$, for any real number r.

If $f(x) = a^x$, then $f'(x) = f'(0)a^x = (\ln a)a^x$, for any positive number a.

If $f(x) = e^x$, then $f'(x) = e^x$, where e is "Euler's e," a number defined by the property

$$\lim_{h \to 0} \frac{e^h - 1}{h} = 0.$$

Exercises

Exercises for Section 3.1, pp 190–192: 1 (the number e), 11, 13, 19, 25, 27, 32, 36, 41 (motion), 44, 50 (parallel lines), 58 (differential equation), 65