Section 3.6: Implicit Differentiation

These notes reflect material from our text, *Calculus, Concepts and Contexts, Third Edition*, by James Stewart, published by Brooks/Cole, Pacific Grove, CA, 2005.

Key points from Stewart, Section 3.6: Implicit differentiation. Derivatives of inverse functions. Derivatives of $arcsin(x), arccos(x), arctan(x)$.

Concepts

Level curves, implicit functions, **implicit differentiation**.

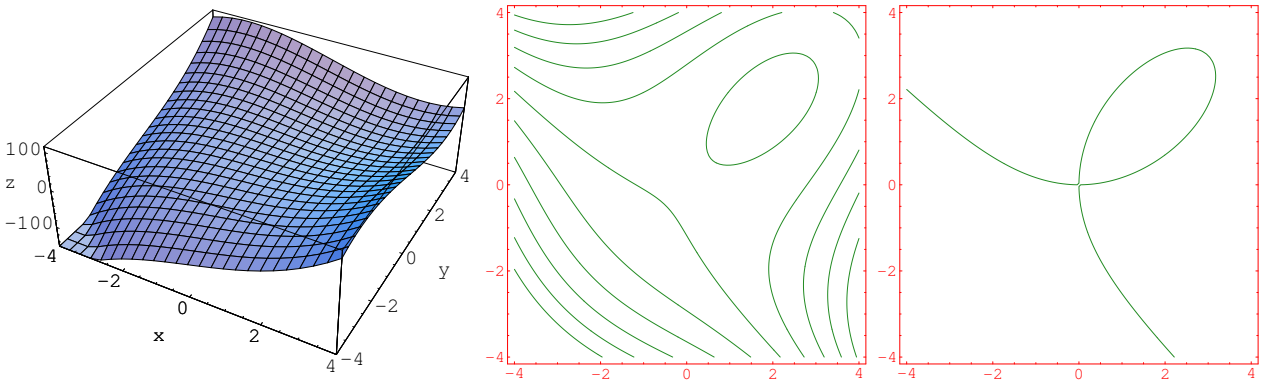

Fig. Folium of Descartes, a level curve of $z = x^3 + y^3 - 6xy$.

The Derivative of an Inverse Function

If both $y = f(x)$ and $x = f^{-1}(y)$ are differentiable functions, then the *derivative of* f^{-1} at y is

$$(f^{-1})'(y) = \frac{1}{f'(x)} = \frac{1}{f'(f^{-1}(y))}.$$

In the notation of Leibniz,

$$\frac{dy}{dx} = \frac{1}{\frac{dx}{dy}}.$$

The Derivatives of Certain Inverse Functions

Arcsine Function

$$\frac{d}{dx}\big(\arcsin x\big) = \frac{1}{\sqrt{1-x^2}}$$

Arccosine Function

$$\frac{d}{dx}\big(\arccos x\big) = \frac{-1}{\sqrt{1-x^2}}$$

Arctangent Function

$$\frac{d}{dx}\big(\arctan x\big) = \frac{1}{1+x^2}$$

Exercises

Exercises for Section 3.6, pp 238–240: 1, 7 (implicit differentiation), 17 (cardioid), 24 (bouncing wagon), 31, 41 (steepest descent), 42 (isobars), 49 (rotated ellipse)