Section 4.3: Derivatives and the Shapes of Curves

These notes reflect material from our text, *Calculus, Concepts and Contexts, Third Edition*, by James Stewart, published by Brooks/Cole, Pacific Grove, CA, 2005.

Key points from Stewart, Section 4.3: Relationship between increasing/decreasing nature of a function and the sign of its first derivative. Relationship between concavity of a function and the sign of its second derivative.

Critical Point

Let p be a point in the domain of f. If either
$f'(p) = 0$ or
$f'(p)$ does not exist,
then p is a *critical point* of f.

Local Minima and Maxima

Let p be a critical point of f. Then
f has a *local minimum* at p if, near p, the values of f get no smaller than $f(p)$.
f has a *local maximum* at p if, near p, the values of f get no larger than $f(p)$.

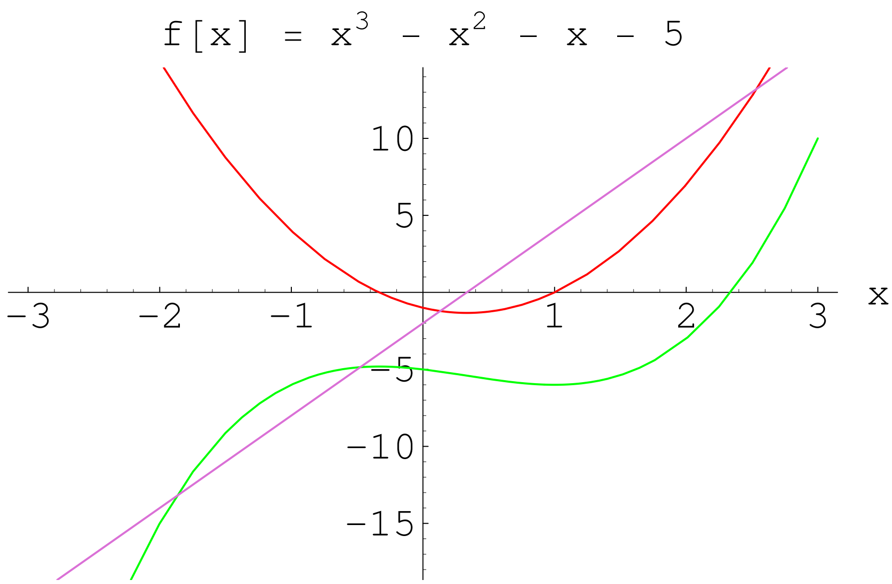

Fig. A function and its derivatives.

First Derivative Test for Local Minima and Maxima

Let p be a critical point of f. If
$f' < 0$ to the left of p and $f' > 0$ to the right of p then f has a local minimum at p.
$f' > 0$ to the left of p and $f' < 0$ to the right of p then f has a local maximum at p.

Global Minima and Maxima

Let f have domain D.

f has a *global minimum* at p if all values of f on D are greater than or equal to $f(p)$.
f has a *global maximum* at p if all values of f on D are less than or equal to $f(p)$.

Inflection Point

A point at which the graph of a function changes concavity is an *inflection point* of f.

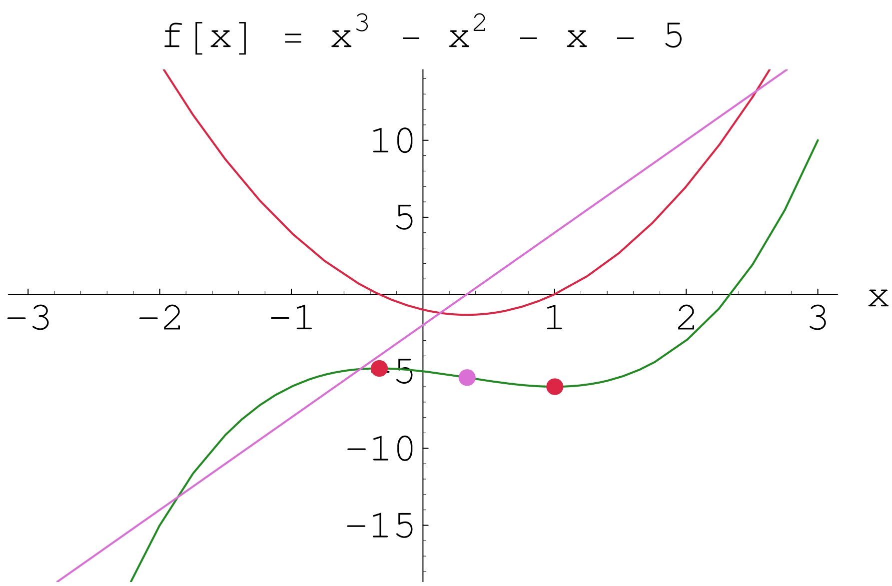

Fig. Including critical points and points of inflection.

Finding Possible Inflection Points

Consider $x = p$. If either

$f''(p) = 0$ or
$f''(p)$ does not exist, then p is a *possible point of inflection* of f.

Second Derivative Test for Local Minima and Maxima

If $f'(p) = 0$ and $f''(p) > 0$ then f has a local minimum at p.
If $f'(p) = 0$ and $f''(p) < 0$ then f has a local maximum at p.

Exercises

Exercises for Section 4.3, pp 286–289: 1, 3, 4, 5, 6, 12, 23, 35, 39, 46 (Coulomb's Law), 47 (VCR's), 48 (bell-shaped curves)