Section 4.8: Newton's Method

These notes reflect material from our text, *Calculus, Concepts and Contexts, Third Edition*, by James Stewart, published by Brooks/Cole, Pacific Grove, CA, 2005.

Key points from Stewart, Section 4.8: Using Newton's method to approximate the zeros of a function.

Concepts

Approximations.

Guessing a value, a, for a root of $f(x)$ and using the tangent line, $y = f(a) + f'(a)(x - a)$, to improve the guess.

The **Newton-Raphson method**,

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}.$$

Accuracy of the Newton-Raphson method. Typically, the number of accurate significant digits in the approximation *doubles* with each iteration, if the initial guess was sufficiently accurate.

Thus, the sequence of approximations, $\{x_n : n \geq 0\}$, *converges* very rapidly. When the method works, it works very well.

The Newton-Raphson method can fail, even dramatically so. When does it succeed?

In many applications, a computer or graphing calculator can provide a reasonably accurate initial guess and the Newton-Raphson method can improve that guess to any desired degree of accuracy.

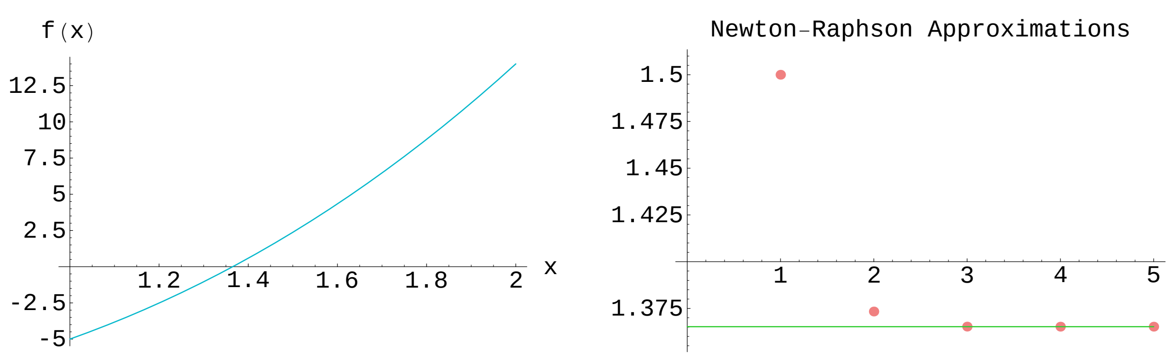

Fig. Convergence of Newton-Raphson approximations.

Exercises

Exercises for Section 4.8, pp 325–327: 1, 4, 8, 14 (roots), 23 (bad initial approximation), 25 (Newton's method fails), 26 (max)