

## Section 5.1: Areas and Distances

These notes reflect material from our text, *Calculus, Concepts and Contexts, Third Edition*, by James Stewart, published by Brooks/Cole, Pacific Grove, CA, 2005.

*Key points from Stewart, Section 5.1: Area problem, distance problem, Riemann sums.*

### Concepts

The **area problem**, finding the area of a region  $R$  that lies under the graph of  $y = f(x)$  between the points  $a$  and  $b$ . The ancient *method of exhaustion* and its modern systematization *Riemann sums*.

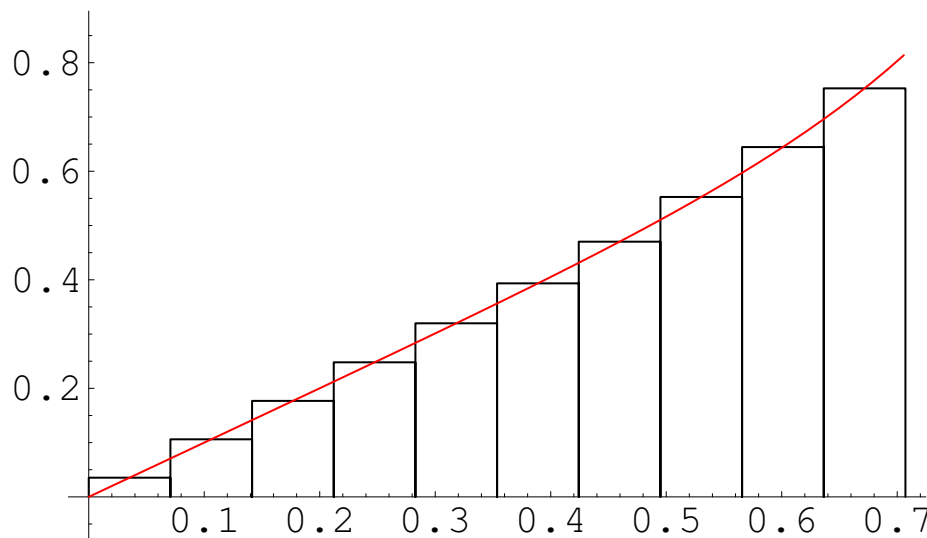
Notation for expressing Riemann sums,

$$\text{area}(R) \approx \sum_{i=1}^n f(x_i^*) \Delta x.$$

Improving the accuracy of a Riemann sum approximation by using more rectangles. Visualizing the errors incurred in Riemann sum approximations for a monotone continuous function  $f$ .

Lefthand and righthand endpoints, midpoints, and other choices for  $x_i^*$ .

The **distance problem**, finding the distance traveled if the velocity function  $v(t)$  is known.



*Fig. Riemann sum using the midpoint rule.*

### Exercises

*Exercises for Section 5.1, pp 352–354:* 2, 6, 9 (CAS), 11 (runner), 13 (oil leak), 14 (Endeavour), 15 (distance traveled), 18 (expression), 19 (region)