

Section 5.5: The Substitution Rule

These notes reflect material from our text, *Calculus, Concepts and Contexts, Third Edition*, by James Stewart, published by Brooks/Cole, Pacific Grove, CA, 2005.

Key points from Stewart, Section 5.5: The substitution method for evaluating integrals.

Concepts

The integration technique known as the **method of substitution** is justified by the identity

$$\int f(g(x)) g'(x) dx = \int f(u) du,$$

a result obtained by integrating the formula for the chain rule. The identity holds if g' is continuous on $[a, b]$ and if f is continuous on the range of g .

The corresponding formula for definite integrals is

$$\int_a^b f(g(x)) g'(x) dx = \int_{g(a)}^{g(b)} f(u) du.$$

A function is **symmetric** if it is either even or odd.

If $f(x)$ is *even* on the interval $[-a, a]$, then $\int_{-a}^a f(x) dx = 2 \int_0^a f(x) dx$.

If $f(x)$ is *odd* on the interval $[-a, a]$, then $\int_{-a}^a f(x) dx = 0$.

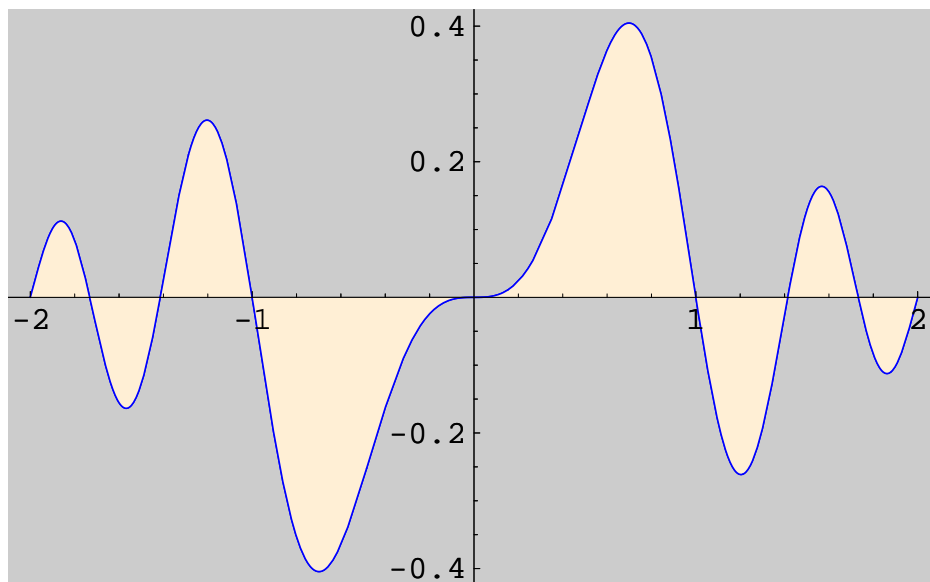


Fig. An odd function.

Exercises

Exercises for Section 5.5, pp 392–393: 3, 11, 13, 19, 23, 33 (indefinite integrals), 45, 53 (definite integrals), 55, 59 (equal areas), 60 (bacteria), 61 (respiratory cycle), 63