Lay Chapter 2, Matrix Algebra

Matrix Operations

Product of Matrices

```
a = {{1, 2}, {3, 4}};
b = {{5, 6}, {7, 8}};

a.b;
% // MatrixForm
```

$$\begin{pmatrix} 19 & 22 \\ 43 & 50 \end{pmatrix}$$

Powers of Matrices

```
a = {{2, 0}, {0, 3}};

MatrixPower[a, 4];
% // MatrixForm
```

$$\begin{pmatrix} 16 & 0 \\ 0 & 81 \end{pmatrix}$$

Transpose of a Matrix

```
a = {{1, 2}, {3, 4}};

Transpose[a];
% // MatrixForm
```

$$\begin{pmatrix} 1 & 3 \\ 2 & 4 \end{pmatrix}$$

Special Matrices

```
IdentityMatrix[3];
% // MatrixForm
```

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

```
DiagonalMatrix[{1, 2, 3}];
% // MatrixForm
```

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{pmatrix}$$

```
<< LinearAlgebra`MatrixManipulation`
```

```
ZeroMatrix[2, 3];
% // MatrixForm
```

$$\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

Matrix Equation A.x == b

Matrix Equation A.x == b

```
Clear[a, x, x1, x2, b, eqns];

a = ((1  2)
     (3  4));
b = {5, 6};

x = {x1, x2};

eqn = {a.x == b};

Print[a // MatrixForm, ".", x // MatrixForm, " = ", b // MatrixForm]

Solve[eqn, {x1, x2}] // Flatten
```

$$\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} \cdot \begin{pmatrix} x1 \\ x2 \end{pmatrix} = \begin{pmatrix} 5 \\ 6 \end{pmatrix}$$

$$\left\{x1 \to -4,\ x2 \to \frac{9}{2}\right\}$$

Matrix Generator

makeMatrix

Let's write a procedure which creates matrices.

```
makeMatrix[n_, m_] := Table[Random[Integer, {-9, 9}], {n}, {m}]
```

Generate a few matrices.
New matrices will be generated each time these cells are evaluated.

```
Clear[a, b, c];

a = makeMatrix[2, 2];
% // MatrixForm
```

$$\begin{pmatrix} -8 & -9 \\ -5 & 0 \end{pmatrix}$$

```
b = makeMatrix[3, 4];
% // MatrixForm
```

$$\begin{pmatrix} -9 & -4 & 2 & 2 \\ -2 & -8 & -2 & 4 \\ 5 & 3 & 3 & -8 \end{pmatrix}$$

```
c = makeMatrix[4, 1];
% // MatrixForm
```

$$\begin{pmatrix} 3 \\ -2 \\ -9 \\ 9 \end{pmatrix}$$

Inverse of a Square Matrix

Matrix Inverse

```
a = {{0, 1, 2}, {1, 0, 3}, {4, -3, 8}};

aInv = Inverse[a];
% // MatrixForm
```

$$\begin{pmatrix} -\frac{9}{2} & 7 & -\frac{3}{2} \\ -2 & 4 & -1 \\ \frac{3}{2} & -2 & \frac{1}{2} \end{pmatrix}$$

Check.

```
id = IdentityMatrix[3];

a.aInv == id
aInv.a == id

True
```

```
True
```

Matrix Inverse Via Row Reduction

Use row reduction to find the inverse of the matrix a.

```
<< LinearAlgebra`MatrixManipulation`
```

```
Clear[a, id, aug, rr, aInv];

a = {{0, 1, 2}, {1, 0, 3}, {4, -3, 8}};
id = IdentityMatrix[3];

aug = AppendRows[a, id];
% // MatrixForm
```

$$\begin{pmatrix} 0 & 1 & 2 & 1 & 0 & 0 \\ 1 & 0 & 3 & 0 & 1 & 0 \\ 4 & -3 & 8 & 0 & 0 & 1 \end{pmatrix}$$

```
rr = RowReduce[aug];
% // MatrixForm
```

$$\begin{pmatrix} 1 & 0 & 0 & -\frac{9}{2} & 7 & -\frac{3}{2} \\ 0 & 1 & 0 & -2 & 4 & -1 \\ 0 & 0 & 1 & \frac{3}{2} & -2 & \frac{1}{2} \end{pmatrix}$$

```
aInv = TakeColumns[rr, -3];
% // MatrixForm
```

$$\begin{pmatrix} -\frac{9}{2} & 7 & -\frac{3}{2} \\ -2 & 4 & -1 \\ \frac{3}{2} & -2 & \frac{1}{2} \end{pmatrix}$$

Check.

```
a.aInv == id
aInv.a == id
```

```
True
```

```
True
```

Block Matrices

Building a Block Matrix

BlockMatrix maps AppendRows across each row of its argument, then applies AppendColumn to the result.

```
<< LinearAlgebra`MatrixManipulation`
```

```
a1 = {{1, 2}, {3, 5}};
a2 = {{2}};
a3 = {{7, 8}, {5, 6}};

z23 = ZeroMatrix[2, 3];
z12 = ZeroMatrix[1, 2];

b = BlockMatrix[
   {{a1, z23},
    {z12, a2, z12},
    {z23, a3}}];
% // MatrixForm
```

$$\begin{pmatrix} 1 & 2 & 0 & 0 & 0 \\ 3 & 5 & 0 & 0 & 0 \\ 0 & 0 & 2 & 0 & 0 \\ 0 & 0 & 0 & 7 & 8 \\ 0 & 0 & 0 & 5 & 6 \end{pmatrix}$$

Extracting a Block

```
Take[b, {3, 5}, {3, 5}];
% // MatrixForm
```

$$\begin{pmatrix} 2 & 0 & 0 \\ 0 & 7 & 8 \\ 0 & 5 & 6 \end{pmatrix}$$

An alternate method.

```
rows = Range[3, 5];
cols = Range[3, 5];

b[[rows, cols]];
% // MatrixForm
```

$$\begin{pmatrix} 2 & 0 & 0 \\ 0 & 7 & 8 \\ 0 & 5 & 6 \end{pmatrix}$$