Lay Chapter 3, Determinants

Determinants

Let's write a procedure which creates matrices.

```
makeMatrix[n_, m_] := Table[Random[Integer, {-9, 9}], {n}, {m}]
```

Now generate a few square matrices and calculate their determinants

```
Clear[a, b, c];

a = makeMatrix[2, 2];
% // MatrixForm
Det[a]
```

$$\begin{pmatrix} 4 & -3 \\ 0 & 8 \end{pmatrix}$$

```
32
```

```
b = makeMatrix[3, 3];
% // MatrixForm
Det[b]
```

$$\begin{pmatrix} 7 & 0 & 9 \\ 6 & 0 & 3 \\ 4 & 7 & -5 \end{pmatrix}$$

```
231
```

```
c = makeMatrix[4, 4];
% // MatrixForm
Det[c]
```

$$\begin{pmatrix} 1 & -4 & -7 & -7 \\ 0 & -9 & -8 & -9 \\ 6 & 9 & -6 & -6 \\ 9 & 0 & -9 & -1 \end{pmatrix}$$

```
-966
```

Determinants in Analytic Geometry

■ Equation of a Line through Two Points

Use a determinant to find the equation of the line which passes through P(0,1) and Q(1,2).
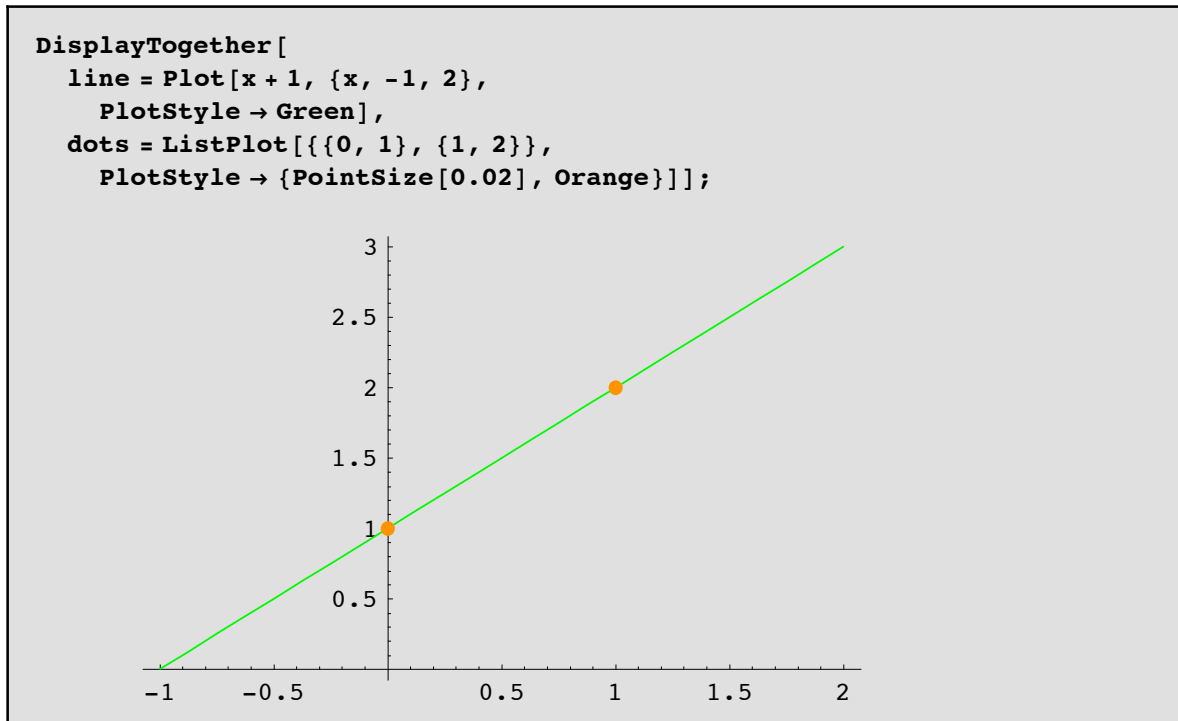See Lay's Case Study for chapter 3, "**Determinants in Analytic Geometry**."

```
Clear[m, x, y];

m = ( x  y  1
      0  1  1 );
      1  2  1

Det[m] == 0

-1 - x + y == 0
```

```
<< Graphics`Graphics`
```

```
DisplayTogether[
  line = Plot[x + 1, {x, -1, 2},
    PlotStyle → Green],
  dots = ListPlot[{{0, 1}, {1, 2}},
    PlotStyle → {PointSize[0.02], Orange}]];
```


■ Equation of a Circle through Three Points

Use a determinant to find the equation of the circle which passes through P(0,1), Q(1,2), and R(2,4).
See Lay's Case Study for chapter 3, "**Determinants in Analytic Geometry**."

```
Clear[m, x, y, r, c, d];

       ⎛ x² + y²   x   y   1 ⎞
m =    ⎜    1      0   1   1 ⎟ ;
       ⎜    5      1   2   1 ⎟
       ⎝   20      2   4   1 ⎠

Det[m] == 0

10 + 7 x + x² - 11 y + y² == 0
```

Find the standard equation for this circle by "completing the squares."

```
std = (x + 7 / 2)² + (y - 11 / 2)² - r² // Expand
rSoln = Solve[Det[m] == std, r]
```
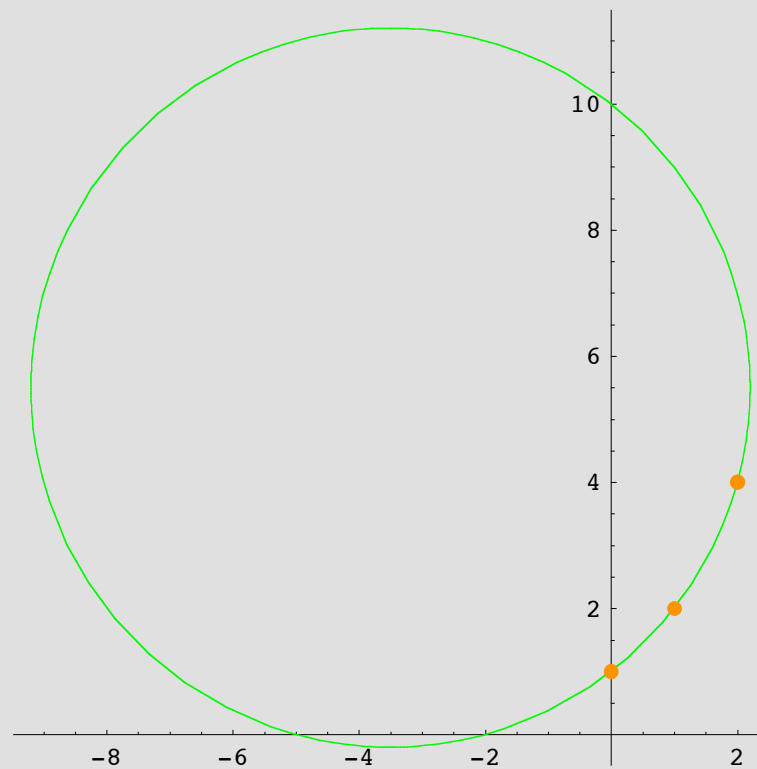
$$\frac{85}{2} - r^2 + 7\,x + x^2 - 11\,y + y^2$$

$$\left\{\left\{r \to -\sqrt{\frac{65}{2}}\,\right\}, \left\{r \to \sqrt{\frac{65}{2}}\,\right\}\right\}$$

```
ans = (x + 7 / 2)² + (y - 11 / 2)² == 65 / 2;
```

```
r = √(65 / 2) ;
c = -7 / 2;
d = 11 / 2;

DisplayTogether[
  circle = ParametricPlot[{c + r Cos[θ], d + r Sin[θ]}, {θ, 0, 2 π},
    PlotStyle → Green, AspectRatio → 1],
  dots = ListPlot[{{0, 1}, {1, 2}, {2, 4}},
    PlotStyle → {PointSize[0.02], Orange}]];
```

■ Equation of a Plane through Three Points

Use a determinant to find the equation of the plane which passes through P(0,1,0), Q(1,2,1), and R(1,3,5).
See Lay's Case Study for chapter 3, "**Determinants in Analytic Geometry**."

```
Clear[a, x, y, z];

a = ( x  y  z  1 )
    ( 0  1  0  1 )
    ( 1  2  1  1 );
    ( 1  3  5  1 )

Det[a] == 0

4 + 3 x - 4 y + z == 0
```

```
Plot3D[-3 x + 4 y - 4, {x, -2, 2}, {y, -2, 2},
  AxesLabel → {x, y, z}];
```