Lay Chapter 4,
Vector Spaces

dim(domain) == dim(kernel) + dim(range)

Consider a certain 3x4 matrix.

```
a = (1 2 3 4
     0 1 2 3
     0 0 1 2);
```

The matrix a can be interpreted as the matrix of a linear map,
and the dimensions of certain vector spaces associated with that linear map will satisfy a well-known relationship.

```
(* dim (domain) == dim (kernel) + dim (range) *)

Last[Dimensions[a]] == Length[NullSpace[a]] + MatrixRank[a]

True
```

Let's write some procedures which will make that relationship more transparent.

```
dimDomain[a_] := Last[Dimensions[a]]
dimKernel[a_] := Length[NullSpace[a]]
dimRange[a_] := MatrixRank[a]
```

Test those procedures on the matrix a.

```
dimDomain[a]
dimKernel[a]
dimRange[a]

dimDomain[a] == dimKernel[a] + dimRange[a]

4
```

```
1
```

```
3
```

```
True
```

Basis for the Kernel of a Linear Map

Consider a certain 2x4 matrix.

$$a = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 0 & 1 & 2 & 3 \end{pmatrix};$$

The matrix a can be interpreted as the matrix of a linear map,
We seek a basis for the null space of this linear map.

```
NullSpace[a];
% // MatrixForm
```

$$\begin{pmatrix} 2 & -3 & 0 & 1 \\ 1 & -2 & 1 & 0 \end{pmatrix}$$