Gram-Schmidt

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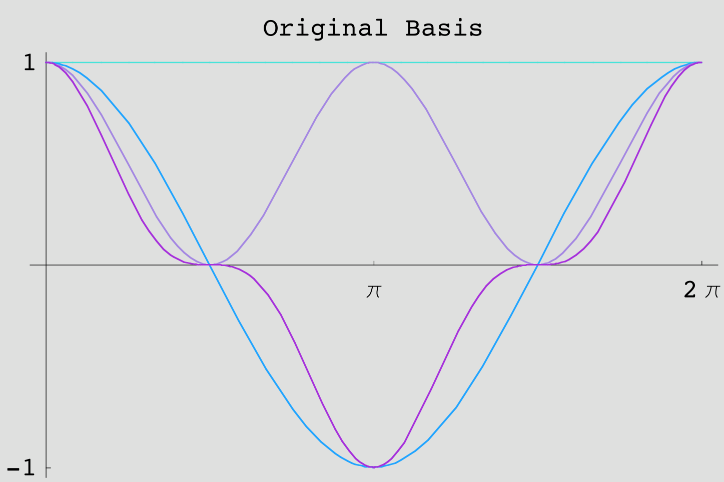
Lay 6.7.28

Find an orthogonal basis for the subspace of C[0,2π] spanned by $\{1, \text{Cos}[t], \text{Cos}[t]^2, \text{Cos}[t]^3\}$.

```
Clear[t, a0, a1, a2, a3, b0, b1, b2,
 b3, originalBasis, orthogonalBasis, prod]

a0[t_] = 1;
a1[t_] = Cos[t];
a2[t_] = Cos[t]^2;
a3[t_] = Cos[t]^3;

originalBasis = {a0[t], a1[t], a2[t], a3[t]};

prod[f_, g_] := ∫_0^(2π) f g ⅆt
```

```
<< Graphics`Colors`
```

```
Plot[Evaluate[originalBasis], {t, 0, 2 π},
  PlotLabel -> "Original Basis",
  Ticks → {{0, π, 2 π}, {-1, 0, 1}},
  PlotStyle → {Turquoise, DodgerBlue, MediumPurple, DarkViolet}];
```



```
b0[t_] = a0[t]
```

1

$$b1[x\_] = a1[t] - \frac{prod[a1[t], b0[t]]}{prod[b0[t], b0[t]]} b0[t]$$

Cos[t]

$$b2[x\_] = a2[t] - \frac{prod[a2[t], b0[t]]}{prod[b0[t], b0[t]]} b0[t] - \frac{prod[a2[t], b1[t]]}{prod[b1[t], b1[t]]} b1[t] \text{ // Simplify}$$

$\frac{1}{2}$ Cos[2 t]

$$b3[x\_] = a3[t] - \frac{prod[a3[t], b0[t]]}{prod[b0[t], b0[t]]} a0[t] - \frac{prod[a3[t], b1[t]]}{prod[b1[t], b1[t]]} a1[t] - \frac{prod[a3[t], b2[t]]}{prod[b2[t], b2[t]]} b2[t] \text{ // Simplify}$$

$\frac{1}{4}$ Cos[3 t]

```
orthogonalBasis = {b0[x], b1[x], b2[x], b3[x]}
```

{1, Cos[t], $\frac{1}{2}$ Cos[2 t], $\frac{1}{4}$ Cos[3 t]}

```
Plot[Evaluate[orthogonalBasis], {t, 0, 2 π},
  PlotLabel -> "Orthogonal Basis",
  PlotStyle → {Turquoise, DodgerBlue, MediumPurple, DarkViolet}];
```