Lay Chapter 7, Spectral Decomposition

Consider the symmetric matrix a
(Lay 7.1, Example 4)

```
Clear[a];

a = (7  2
     2  4);

a == Transpose[a]

True
```

Calculate the eigenvalues and eigenvectors of a.

```
{evals, evecs} = Eigensystem[a]

{{8, 3}, {{2, 1}, {-1, 2}}}
```

Construct the orthogonal diagonalization of a.

```
p = Table[evecs[[k]] / Norm[evecs[[k]]],
    {k, 1, 2}] // Transpose;
% // MatrixForm
invp = Inverse[p];

diagλ = DiagonalMatrix[evals];
% // MatrixForm

a == p.diagλ.invp
```

$$\begin{pmatrix} \frac{2}{\sqrt{5}} & -\frac{1}{\sqrt{5}} \\ \frac{1}{\sqrt{5}} & \frac{2}{\sqrt{5}} \end{pmatrix}$$

$$\begin{pmatrix} 8 & 0 \\ 0 & 3 \end{pmatrix}$$

```
True
```

Construct a spectral decomposition of a.

```
<< LinearAlgebra`MatrixManipulation`
```

```
u1 = TakeColumns[p, {1, 1}];
% // MatrixForm

u2 = TakeColumns[p, {2, 2}];
% // MatrixForm
```

$$\begin{pmatrix} \frac{2}{\sqrt{5}} \\ \frac{1}{\sqrt{5}} \end{pmatrix}$$

$$\begin{pmatrix} -\frac{1}{\sqrt{5}} \\ \frac{2}{\sqrt{5}} \end{pmatrix}$$

```
blocks = {u1.Transpose[u1], u2.Transpose[u2]};
Map[MatrixForm, %]
```

$$\left\{\begin{pmatrix} \frac{4}{5} & \frac{2}{5} \\ \frac{2}{5} & \frac{1}{5} \end{pmatrix}, \begin{pmatrix} \frac{1}{5} & -\frac{2}{5} \\ -\frac{2}{5} & \frac{4}{5} \end{pmatrix}\right\}$$

```
Print[a // MatrixForm, "==", evals[[1]], blocks[[1]] // MatrixForm,
  "+", evals[[2]], blocks[[2]] // MatrixForm];
```

$$\text{spectralDecomposition} = \sum_{k=1}^{2} \text{evals[[k]] blocks[[k]]};$$

```
a == spectralDecomposition
```

$$\begin{pmatrix} 7 & 2 \\ 2 & 4 \end{pmatrix} == 8 \begin{pmatrix} \frac{4}{5} & \frac{2}{5} \\ \frac{2}{5} & \frac{1}{5} \end{pmatrix} + 3 \begin{pmatrix} \frac{1}{5} & -\frac{2}{5} \\ -\frac{2}{5} & \frac{4}{5} \end{pmatrix}$$

```
True
```