Lay, Section 4.1: Vector Spaces and Subspaces

These notes reflect material from our text, *Linear Algebra and Its Applications, Third Edition*, by David C. Lay, published by Addison Wesley, Boston, 2003.

Definitions

(1) vector space V

(2) subspace U of a vector space V

(3) linear combination of vectors, $a_1 v_1 + \ldots + a_n v_n$

(4) span of a finite set of vectors, $Span(v_1, \ldots, v_n)$

(5) subspace spanned by a set of vectors

Examples of Vector Spaces

(1) the canonical vector spaces, $\mathcal{R}^n$

(2) polynomials in the variable X with real coefficients, $\mathcal{P}[X]$

(3) the space of all infinite sequences, $(a_i)_{i=0}^{\infty}$, of real numbers

(4) the space of all solutions of the homogeneous differential equation $X'' + X = 0$

(5) the space of all $m \times n$ matrices over the real numbers, $\mathcal{M}_{m \times n}(\mathcal{R})$

(6) $\mathcal{L}(U, V)$, the space of all linear maps from the vector space U to the vector space V

(7) $\mathcal{C}[a, b]$, the space of all continuous, real-valued functions defined on the interval $[a, b]$

Results

Theorem. *The span of a set of vectors of a vector space V is a subspace of V.*

Case Study: Space Flight and Control Systems

Download the supporting pdf file and Mathematica notebook from the Lay Linear Algebra web site.

Exercises

We will solve some of the following exercises as a community project in class today. Finish these solutions as homework exercises, write them up carefully and clearly, and hand them in at the beginning of class next Friday. You are encouraged to use a computer algebra system whenever appropriate.

Exercises for Lay, Section 4.1, pp 223–225: 5, 7, 9, 15, 19, 20, 21, 31, 32 ($H \cap K$), 33, 34 ($H + K$)