Lay, Section 4.2: Null Spaces, Column Spaces, and Linear Transformations

These notes reflect material from our text, *Linear Algebra and Its Applications, Third Edition*, by David C. Lay, published by Addison Wesley, Boston, 2003.

Definitions

(1) null space of a matrix $M \in \mathcal{M}_{m\times n}(\mathcal{R})$

(2) column space of a matrix $M \in \mathcal{M}_{m\times n}(\mathcal{R})$

(3) linear transformation $T \in \mathcal{L}(V, W)$

(4) kernel and image of a linear transformation $T \in \mathcal{L}(V, W)$

Results

Let $M \in \mathcal{M}_{m\times n}(\mathcal{R})$.

Theorem. *The null space of a matrix $M \in \mathcal{M}_{m\times n}(\mathcal{R})$ is a subspace of $\mathcal{R}^n$.*

Theorem. *$Null\ M = \{0\}$ iff the equation $Mx = 0$ has only the trivial solution.*

Theorem. *$Null\ M = \{0\}$ iff the linear transformation $x \mapsto Mx$ is one-to-one.*

Theorem. *The column space of a matrix $M \in \mathcal{M}_{m\times n}(\mathcal{R})$ is a subspace of $\mathcal{R}^m$.*

Theorem. *$Col\ M = \mathcal{R}^m$ iff the equation $Mx = b$ has a solution for every $b \in \mathcal{R}^m$.*

Theorem. *$Col\ M = \mathcal{R}^m$ iff the linear transformation $x \mapsto Mx$ maps $\mathcal{R}^n$ onto $\mathcal{R}^m$.*

Algorithms

Algorithms for the explicit determination of spanning sets for the null space and column space of a given matrix $M \in \mathcal{M}_{m\times n}(\mathcal{R})$

Exercises

We will solve some of the following exercises as a community project in class today. Finish these solutions as homework exercises, write them up carefully and clearly, and hand them in at the beginning of class next Friday. You are encouraged to use a computer algebra system whenever appropriate.

Exercises for Lay, Section 4.2, pp 234–236: 1, 3, 9, 15, 19, 23, 33, 34, 39, 40 ($H \cap K$)