Lay, Section 4.3: Linearly Independent Sets; Bases

These notes reflect material from our text, *Linear Algebra and Its Applications, Third Edition*, by David C. Lay, published by Addison Wesley, Boston, 2003.

Definitions

(1) linearly independent vectors in a vector space V

(2) linear dependence relation

(3) basis of a vector space V

Results

Definition. *A list of vectors* $\{v_1, \ldots, v_p\}$ *in a vector space* V *is* **linearly independent** *if the equation*

$$a_1 v_1 + \ldots + a_p v_p = 0$$

has only the trivial solution,

$$a_1 = \ldots = a_p = 0.$$

A list of vectors which is not linearly independent is **linearly dependent.**

Theorem. *(Linear Dependence Lemma). If the list of vectors* $\{v_1, \ldots, v_p\}$ *in the vector space* V *is linearly dependent and* $v_1 \neq 0$*, then one of the vectors in the list is a linear combination of the previous vectors; thus, for some index* k *and appropriate coefficients* $a_1, \ldots, a_{k-1}$,

$$v_k = a_1 v_1 + \ldots + a_{k-1} v_{k-1}.$$

Spanning Set Theorem. *(Lay, Theorem 5, p. 239).*

Definition. *A list of vectors* $\beta = \{v_1, \ldots, v_p\}$ *in a subspace* U *of a vector space* V *is a* **basis** *for* U *if* β *is a linearly independent set which spans* U*.*

Two characterizations of a basis for the vector space V:
(1) A basis is a spanning set that is as small as possible.
(2) A basis is a linearly independent set that is as large as possible.

Algorithms

Algorithms for the explicit determination of bases for $Null\ A$ and $Col\ A$, for a matrix $A \in \mathcal{M}_{m\times n}(\mathcal{R})$.

Exercises

We will solve some of the following exercises as a community project in class today. Finish these solutions as homework exercises, write them up carefully and clearly, and hand them in at the beginning of class next Friday. You are encouraged to use a computer algebra system whenever appropriate.

Exercises for Lay, Section 4.3, pp 243–245: 3, 9, 12, 13, 19, 37, 38 (sines and cosines)