Lay, Section 4.4: Coordinate Systems

These notes reflect material from our text, *Linear Algebra and Its Applications, Third Edition*, by David C. Lay, published by Addison Wesley, Boston, 2003.

Definitions

(1) $[v]_\beta$, coordinates of a vector $v \in V$ with respect to a basis β of V

(2) the coordinate mapping $\Phi : v \mapsto [v]_\beta$ determined by a basis β of V

(3) coordinate lattice in $\mathcal{R}^2$ or $\mathcal{R}^3$

(4) change of coordinates matrix

(5) isomorphic vector spaces

Results

Theorem. *Let V be an n-dimensional vector space over $\mathcal{R}$, and let β be a basis for V. Then the coordinate mapping $\Phi : V \to \mathcal{R}^n$ given by $v \mapsto [v]_\beta$ is a vector space isomorphism.*

Corollary. *All vector spaces of dimension n are isomorphic to $\mathcal{R}^n$.*

Exercises

We will solve some of the following exercises as a community project in class today. Finish these solutions as homework exercises, write them up carefully and clearly, and hand them in at the beginning of class next Friday. You are encouraged to use a computer algebra system whenever appropriate.

Exercises for Lay, Section 4.4, pp 253–255: 1, 5, 9, 11, 13, 27, 37, 38 (crystal lattice for titanium)