

Lay, Section 4.5: The Dimension of a Vector Space

These notes reflect material from our text, *Linear Algebra and Its Applications, Third Edition*, by David C. Lay, published by Addison Wesley, Boston, 2003.

Definitions

- (1) finite dimensional vector space V
- (2) infinite dimensional vector space V
- (3) $\beta = (b_1, \dots, b_n)$, a basis of the finite-dimensional vector space V
- (4) $\dim V$, the dimension of a finite-dimensional vector space V
- (5) $\dim \text{Null } A$, the dimension of the null space of the matrix A
- (6) $\dim \text{Col } A$, the dimension of the column space of the matrix A

Results

Theorem. *If U is a subspace of the finite-dimensional vector space V , then $\dim U \leq \dim V$.*

Theorem. *$\dim \text{Null } A$ is the number of free variables in the matrix equation $Ax = 0$.*

Theorem. *$\dim \text{Col } A$ is the number of pivot columns in the matrix A .*

Exercises

We will solve some of the following exercises as a community project in class today. Finish these solutions as homework exercises, write them up carefully and clearly, and hand them in at the beginning of class next Friday. You are encouraged to use a computer algebra system whenever appropriate.

Exercises for Lay, Section 4.5, pp 260–262: 3, 7, 9, 11, 15, 21, 23 (Hermite polynomials), 27 ($\mathcal{P}[X]$), 33 (expanding a linearly independent set to a basis), 34 (cosines)