Lay, Section 4.7: Change of Basis

These notes reflect material from our text, *Linear Algebra and Its Applications, Third Edition*, by David C. Lay, published by Addison Wesley, Boston, 2003.

Definitions

(1) $[v]_\beta$, the coordinates of a vector $v \in V$ with respect to the basis β of V

Results

Theorem (Change of Coordinates Matrix). *Let $\alpha = (a_1, \ldots, a_n)$ and $\beta = (b_1, \ldots, b_n)$ be two bases of the finite-dimensional vector space V. Then there is a unique matrix $P_{\alpha \leftarrow \beta}$ such that*

$$[v]_\alpha = P_{\alpha \leftarrow \beta}[v]_\beta$$

for every $v \in V$. The columns of $P_{\alpha \leftarrow \beta}$ are the α coordinates of the vectors in β. Thus, the j-th column of $P_{\alpha \leftarrow \beta}$ is $[b_j]_\alpha$.

Algorithms

Algorithm for calculating the change of coordinates matrix $P_{\alpha \leftarrow \beta}$,

$$[\alpha \ \beta] \sim [I \ P_{\alpha \leftarrow \beta}]$$

Exercises

We will solve some of the following exercises as a community project in class today. Finish these solutions as homework exercises, write them up carefully and clearly, and hand them in at the beginning of class next Friday. You are encouraged to use a computer algebra system whenever appropriate.

Exercises for Lay, Section 4.7, pp 276–277: 1, 3, 5, 7, 9, 17, 18 ($\cos kt$ and $\cos^k t$)