Lay, Section 4.8: Applications to Difference Equations

These notes reflect material from our text, *Linear Algebra and Its Applications, Third Edition*, by David C. Lay, published by Addison Wesley, Boston, 2003.

Definitions

(1) the space S of discrete-time signals, $\{y_k\}$

(2) linear independence of signals

(3) Casorati matrices

(4) homogeneous and nonhomogeneous recurrence relations (or difference equations) of order n

(5) linear filters, filter coefficients

(6) input, output, and solutions of difference equations

(7) solutions of the form r^k, auxiliary equations, complex roots

(8) solution set of a homogeneous difference equation viewed as the kernel of a linear map

Results

Theorem. *The set of all solutions of the n-th order homogeneous difference equation*

$$y_{k+n} + a_1 y_{k+n-1} + \ldots + a_{n-1} y_{k+1} + a_n y_k = 0 \text{ for all } k$$

is an n-dimensional vector space.

Theorem. *The difference of any two particular solutions of the n-th order nonhomogeneous difference equation*

$$y_{k+n} + a_1 y_{k+n-1} + \ldots + a_{n-1} y_{k+1} + a_n y_k = z_k \text{ for all } k$$

is a solution of the associated homogeneous equation.

Algorithms

Approaches to calculating the solution set for a given homogeneous or nonhomogeneous difference equation

Algorithm for transforming a homogeneous, n-th order difference equation into an equivalent system of first-order difference equations.

Exercises

We will solve some of the following exercises as a community project in class today. Finish these solutions as homework exercises, write them up carefully and clearly, and hand them in at the beginning of class next Friday. You are encouraged to use a computer algebra system whenever appropriate.

Exercises for Lay, Section 4.8, pp 285–287: 1, 3, 5, 9, 13, 19, 20 (cantilevered beam), 21 (moving average), 29 (first-order system)