

Lay, Section 4.9: Applications to Markov Chains

These notes reflect material from our text, *Linear Algebra and Its Applications, Third Edition*, by David C. Lay, published by Addison Wesley, Boston, 2003.

Definitions

- (1) probability vector, x_k
- (2) stochastic matrix, P
- (3) Markov chain, $\{x_k\}_{k=0}^{\infty}$ with $x_{k+1} = Px_k$ for $k = 0, 1, 2, \dots$
- (4) steady-state or equilibrium vector, $Pq = q$
- (5) regular stochastic matrix

Results

Theorem. *If P is a regular stochastic matrix, then P has a unique steady-state or equilibrium vector q with $Pq = q$. Moreover, for any initial state x_0 , the Markov chain $x_{k+1} = Px_k$, for $k = 0, 1, 2, \dots$, converges to q as $k \rightarrow \infty$.*

Exercises

We will solve some of the following exercises as a community project in class today. Finish these solutions as homework exercises, write them up carefully and clearly, and hand them in at the beginning of class next Friday. You are encouraged to use a computer algebra system whenever appropriate.

Exercises for Lay, Section 4.9, pp 296–298: 1, 4, 11, 14, 15 (California demographics), 16 (Hertz rental cars), 17, 18, 19 (stochastic matrices)