Section 5.2: State Variables and Numerical Approximation


Concepts

- Time-state trajectory
- Phase plane

Results

Initial-value problems of the form

\[ y'' + p y' + q y = f, \quad y(t_0) = y_0, \quad y'(t_0) = v_0 \]

can be represented as first-order linear systems,

\[ y' = v, \quad v' = -p v - q y + f, \quad y(t_0) = y_0, \quad v(t_0) = v_0. \]

*Existence and uniqueness theorems* for linear and nonlinear second-order initial-value problems.

*Numerical approximations* for first-order linear systems.

![Time-state trajectory for a damped oscillatory system](image)

*Fig. Time-state trajectory for a damped oscillatory system, $y'' + y' + ty = \sin t$, $y(0) = 1$, $y'(0) = 0$*

Exercises

We will attempt to solve some of the following exercises as a community project in class today. Finish these solutions as homework exercises, write them up carefully and clearly, and hand them in at the beginning of class next Thursday.

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\begin{align*}
\text{DE} & \quad \text{Exercises for Section 5.2, pp 145–147: 1, 3, 7, 11, 12, 13} \\
\text{AMCDE} & \quad \text{Exercises for Section 6.1, p 117–119: 4, 5, 6 (Plotting Solutions)}
\end{align*}
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