Larsen and Marx 2.3: The Probability Function

These notes reflect material from our text, *Introduction to Mathematical Statistics and Its Applications, Fifth Edition*, by Richard J. Larsen and Morris L. Marx, Pearson, ISBN 978-0-321-69394-5, 2012.

Kolmogorov Axioms for a Probability Function

Andrey Nikolaevich Kolmogorov (1903 – 1987) was an eminent Soviet mathematician who was particularly well-known for his contributions to probability.

Let S be a sample space and let P be a function defined from S into the interval $I = [0, 1]$. Then $P\colon S \to [0, 1]$ is a *probability function* if it satisfies the following four **Kolmogorov Axioms**:

Axiom 1 Let A be any event defined over the sample space S. Then $P(A) \geq 0$.

Axiom 2 $P(S) = 1$.

Axiom 3 If A and B are any two mutually exclusive events defined over S, then

$$P(A \cup B) = P(A) + P(B).$$

Axiom 4 If $A_1, A_2, \ldots$ are events defined over S, and if $A_i \cap A_j = \phi$ for all $i \neq j$, then

$$P(\bigcup_{i=1}^{\infty} A_i) = \sum_{i=1}^{\infty} P(A_i).$$

A number of results follow immediately from these axioms. See the six theorems in Larsen and Marx, p.28, and pay careful attention to how each theorem is proved. Such methods are used repeatedly in exercises such as the following.

Exercises from Larsen and Marx, Section 2.3: 1, 2, 5, 9, 15, 18

To hand in for homework:

Homework exercises from Larsen and Marx, Section 2.3: 1, 9, 15, 18