Larsen and Marx 2.4: Conditional Probability

These notes reflect material from our text, *Introduction to Mathematical Statistics and Its Applications, Fifth Edition*, by Richard J. Larsen and Morris L. Marx, Pearson, ISBN 978-0-321-69394-5, 2012.

Conditional Probability

Definition 1 *Let A and B be two events defined over the sample space S, and assume that $P(B) > 0$. The* conditional probability *of A given that B has already occurred is*

$$P(A \mid B) = \frac{P(A \cap B)}{P(B)}.$$

Definition 2 *A set of events $A_1, A_2, \ldots, A_n$ of a sample space S is a* partition of S *if $S = \cup_{i=1}^{n} A_i$ and $A_i \cap A_j = \phi$ for all $i \neq j$.*

Theorem 1 (Law of Total Probability) *If the set of events $A_1, A_2, \ldots, A_n$ forms a partition of a sample space S then for any event B of S,*

$$P(B) = \sum_{i=1}^{n} P(B \mid A_i) P(A_i).$$

Theorem 2 (Bayes' Theorem) *If the set of events $A_1, A_2, \ldots, A_n$ forms a partition of a sample space S, and $P(A_i) > 0$ for $1 \leq i \leq n$, then for any event B of S with $P(B) > 0$,*

$$P(A_j \mid B) = \frac{P(B \mid A_j) P(A_j)}{\sum_{i=1}^{n} P(B \mid A_i) P(A_i)}.$$

Note that the numerator in the formula for Bayes' Theorem is $P(A_j \cap B)$ and the denominator is an expression for $P(B)$ obtained from the Law of Total Probability.

Exercises from Larsen and Marx, Section 2.4: 1, 6, 19, 21, 25, 27, 33, 39, 42, 44, 46

To hand in for homework:

Homework exercises from Larsen and Marx, Section 2.4: 19, 21, 39, 44