

quadratic approximation

Chris Parrish

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references:	
- McElreath, Statistical Rethinking, chap 2, p.40	
- add legend in <code>ggplot2</code>	
- legends in <code>ggplot2</code>	

quadratic approximation

model

$$w \sim \text{Binomial}(9, p)$$
$$p \sim \text{Uniform}(0, 1)$$

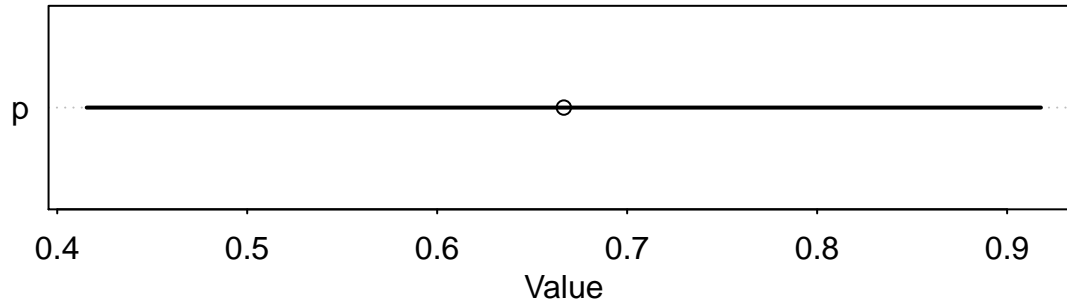
map

```
## R code 2.6
library(rethinking)
globe.qa <- map(
  alist(
    w ~ dbinom(9,p) , # binomial likelihood
    p ~ dunif(0,1)   # uniform prior
  ) ,
  data=list(w=6) )

# display summary of quadratic approximation
precis( globe.qa )
```

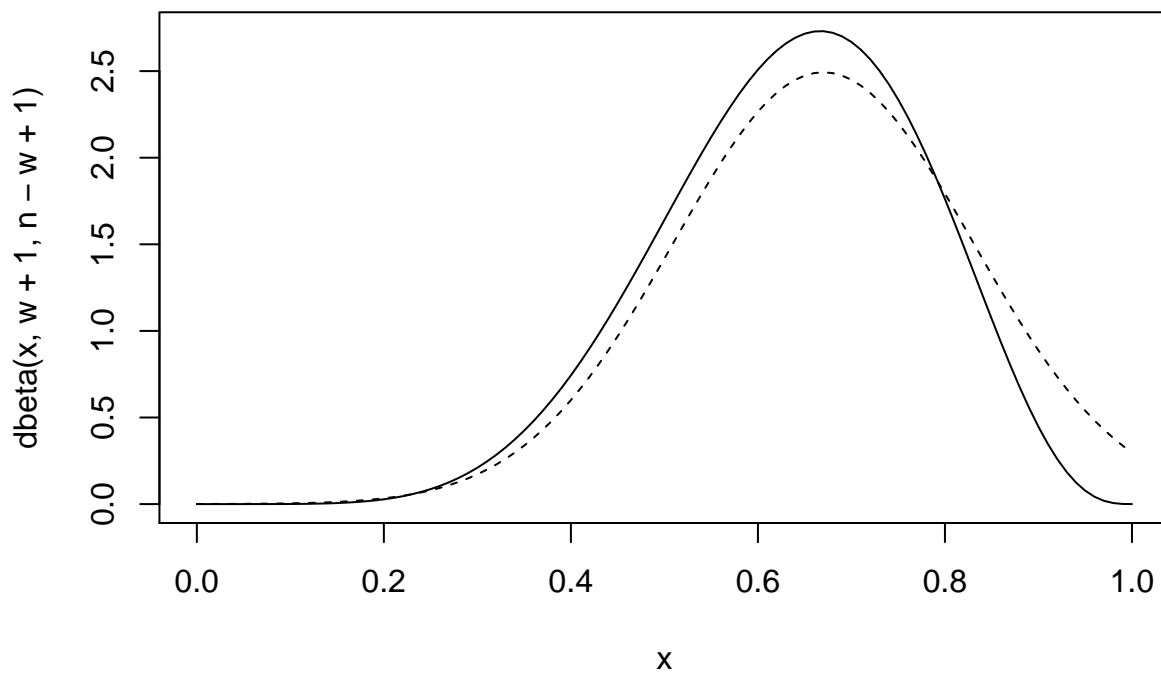
```
## Mean StdDev 5.5% 94.5%
## p 0.67 0.16 0.42 0.92
```

```
plot(precis( globe.qa ))
```



analytical approximation

```
## R code 2.7  
# analytical calculation  
w <- 6  
n <- 9  
curve( dbeta( x , w+1 , n-w+1 ) , from=0 , to=1 )  
# quadratic approximation  
curve( dnorm( x , 0.67 , 0.16 ) , lty=2 , add=TRUE )
```



double the amount of data

```
globe.qa <- map(  
  alist(  
    w ~ dbinom(9 * 2, p) , # binomial likelihood  
    p ~ dunif(0, 1)       # uniform prior  
  ) ,
```

```

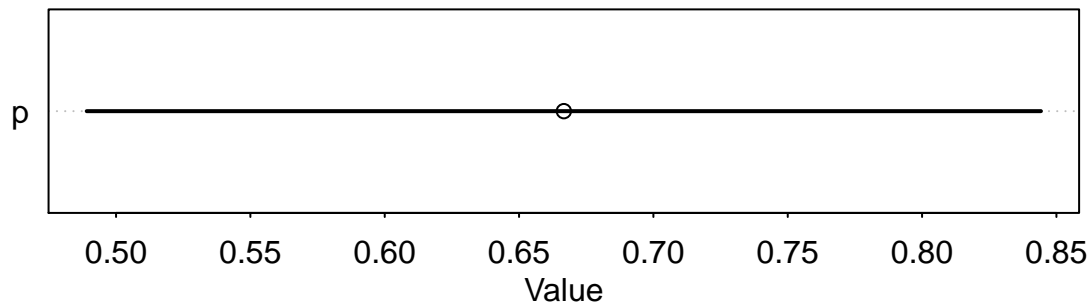
data=list(w=6 * 2 )

# display summary of quadratic approximation
precis( globe.qa )

##   Mean StdDev 5.5% 94.5%
## p 0.67   0.11 0.49  0.84

plot(precis( globe.qa ))

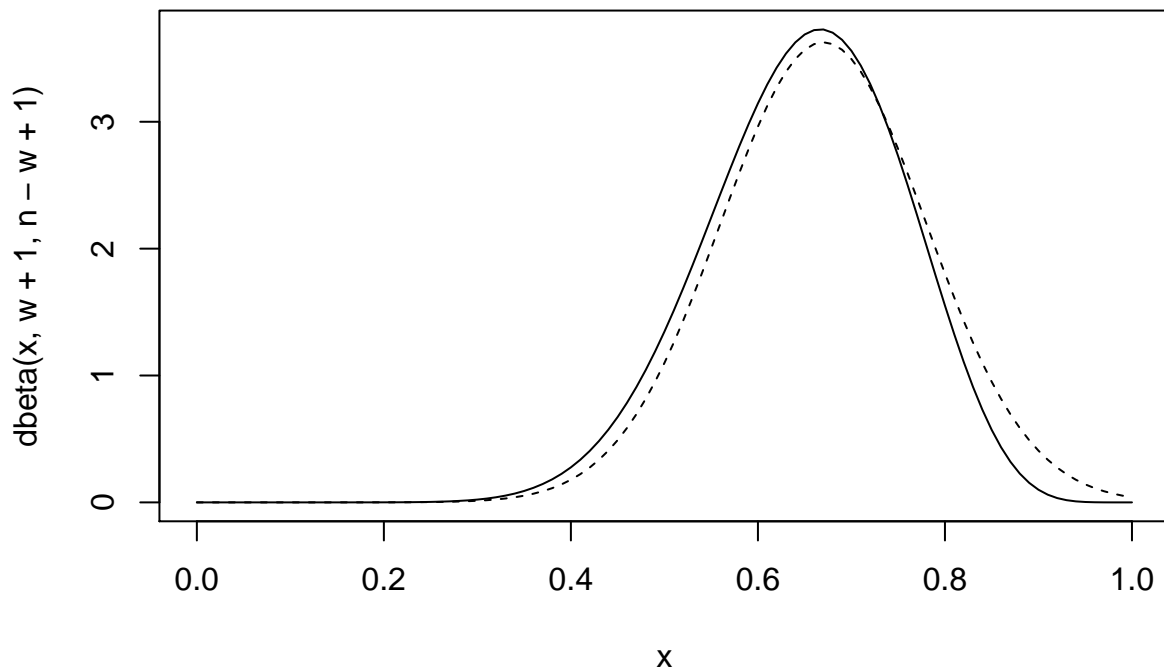
```



```

# analytical calculation
w <- 6 * 2
n <- 9 * 2
curve( dbeta( x , w+1 , n-w+1 ) , from=0 , to=1 )
# quadratic approximation
curve( dnorm( x , 0.67 , 0.11 ) , lty=2 , add=TRUE )

```



triple the amount of data

```

globe.qa <- map(
  alist(
    w ~ dbinom(9 * 3, p) , # binomial likelihood

```

```

    p ~ dunif(0, 1)      # uniform prior
  ) ,
  data=list(w=6 * 3) )

```

```

# display summary of quadratic approximation
precis( globe.qa )

```

```

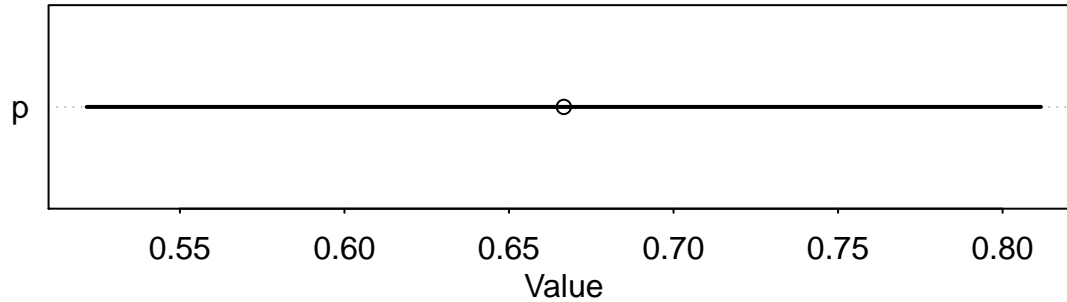
## Mean StdDev 5.5% 94.5%
## p 0.67  0.09 0.52  0.81

```

```

plot(precis( globe.qa ))

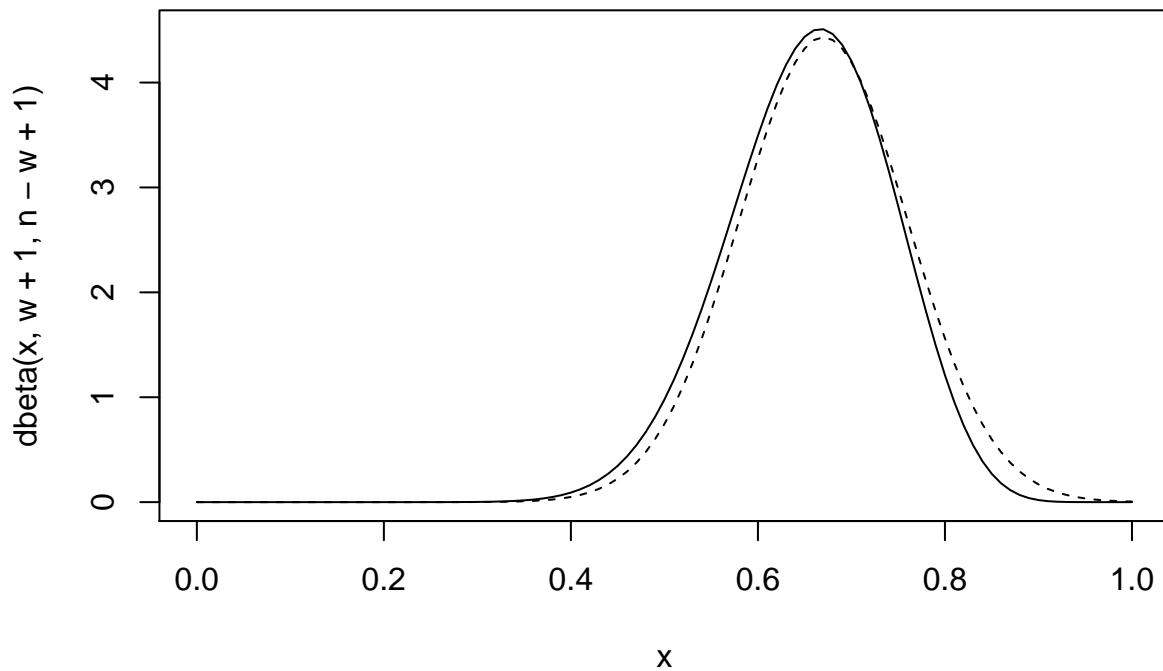
```



```

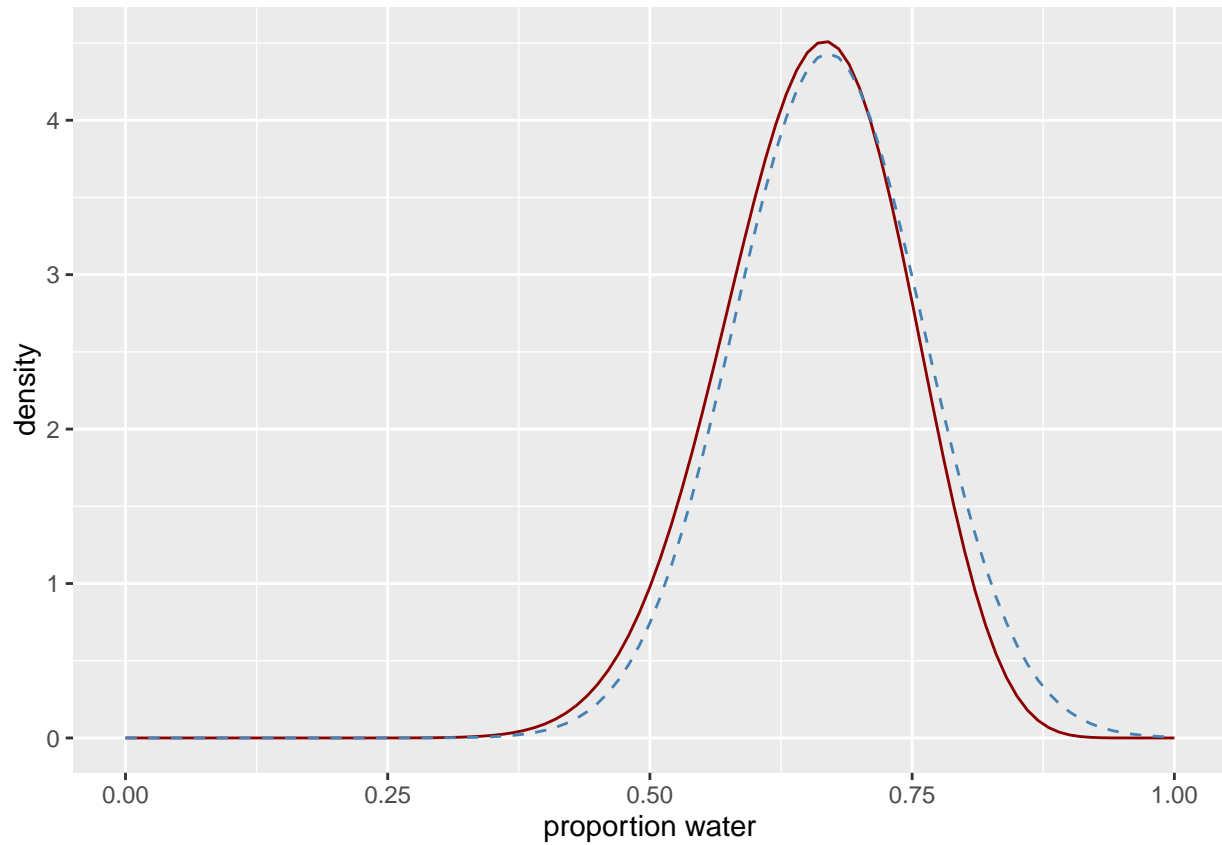
# analytical calculation
w <- 6 * 3
n <- 9 * 3
curve( dbeta( x , w+1 , n-w+1 ) , from=0 , to=1 )
# quadratic approximation
curve( dnorm( x , 0.67 , 0.09 ) , lty=2 , add=TRUE )

```



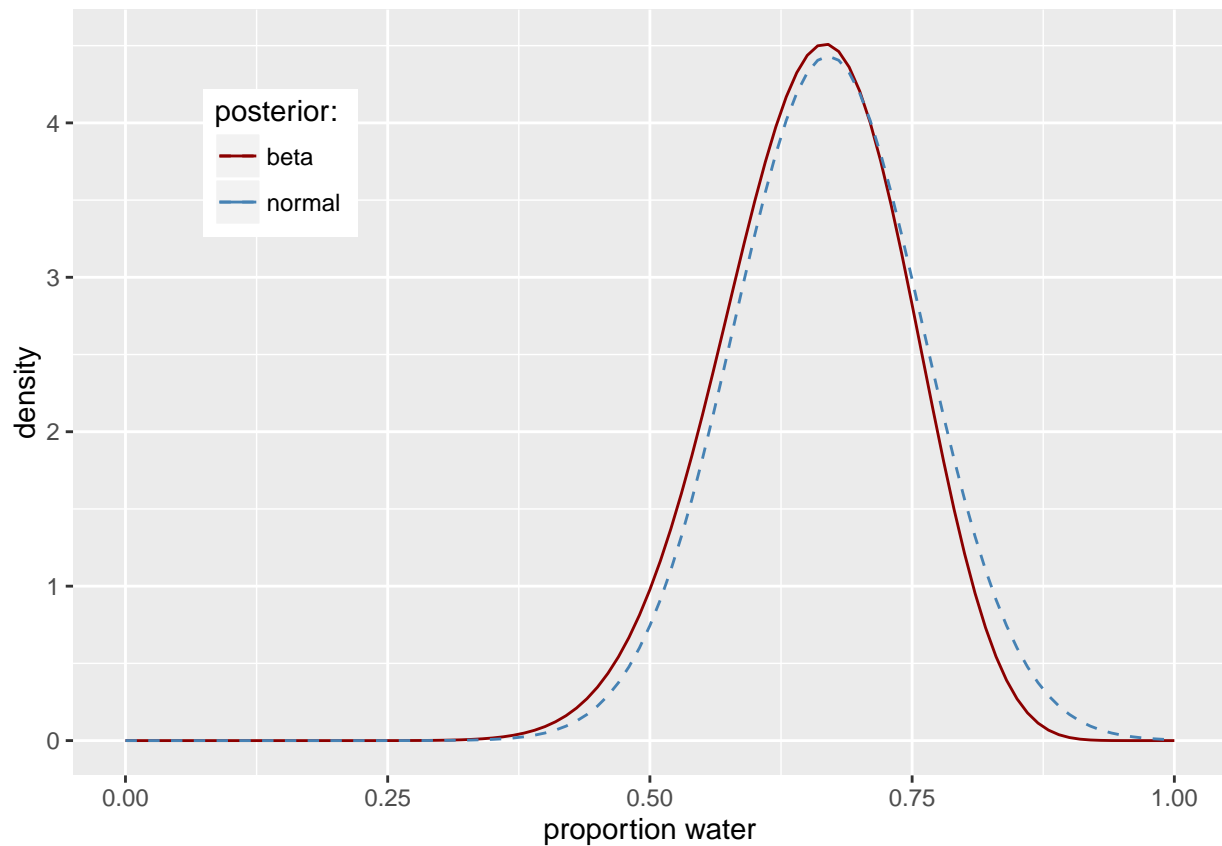
display with ggplot2

```
data <- data.frame(x = c(0, 1), y = c(0, 5))
library(ggplot2)
ggplot(data, aes(x, y)) +
  stat_function(fun = dbeta, args = list(shape1 = w+1, shape2 = n-w+1),
              color = "darkred") +
  stat_function(fun = dnorm, args = list(mean = 0.67, sd = 0.09),
              color = "steelblue", lty = 2) +
  labs(x = "proportion water" , y = "density")
```



add legend

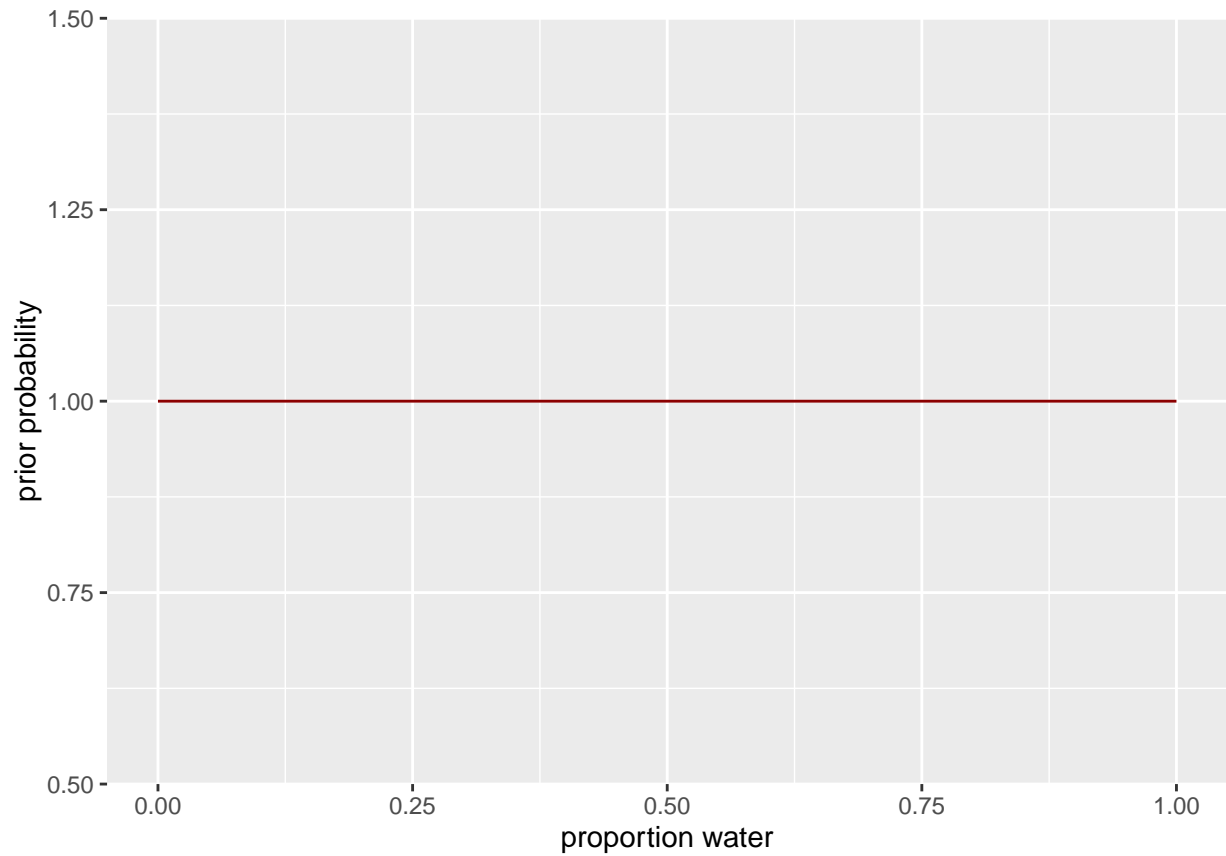
```
data <- data.frame(x = c(0, 1), y = c(0, 5))
plot.posterior.approx <- ggplot(data, aes(x, y)) +
  stat_function(fun = dbeta, args = list(shape1 = w+1, shape2 = n-w+1),
    aes(color = "beta")) +
  stat_function(fun = dnorm, args = list(mean = 0.67, sd = 0.09),
    lty = 2, aes(color = "normal")) +
  scale_color_manual(name = "posterior:",
    values = c("beta" = "darkred", "normal" = "steelblue")) +
  theme(legend.position = c(0.18, 0.8)) +
  labs(x = "proportion water" , y = "density")
plot.posterior.approx
```



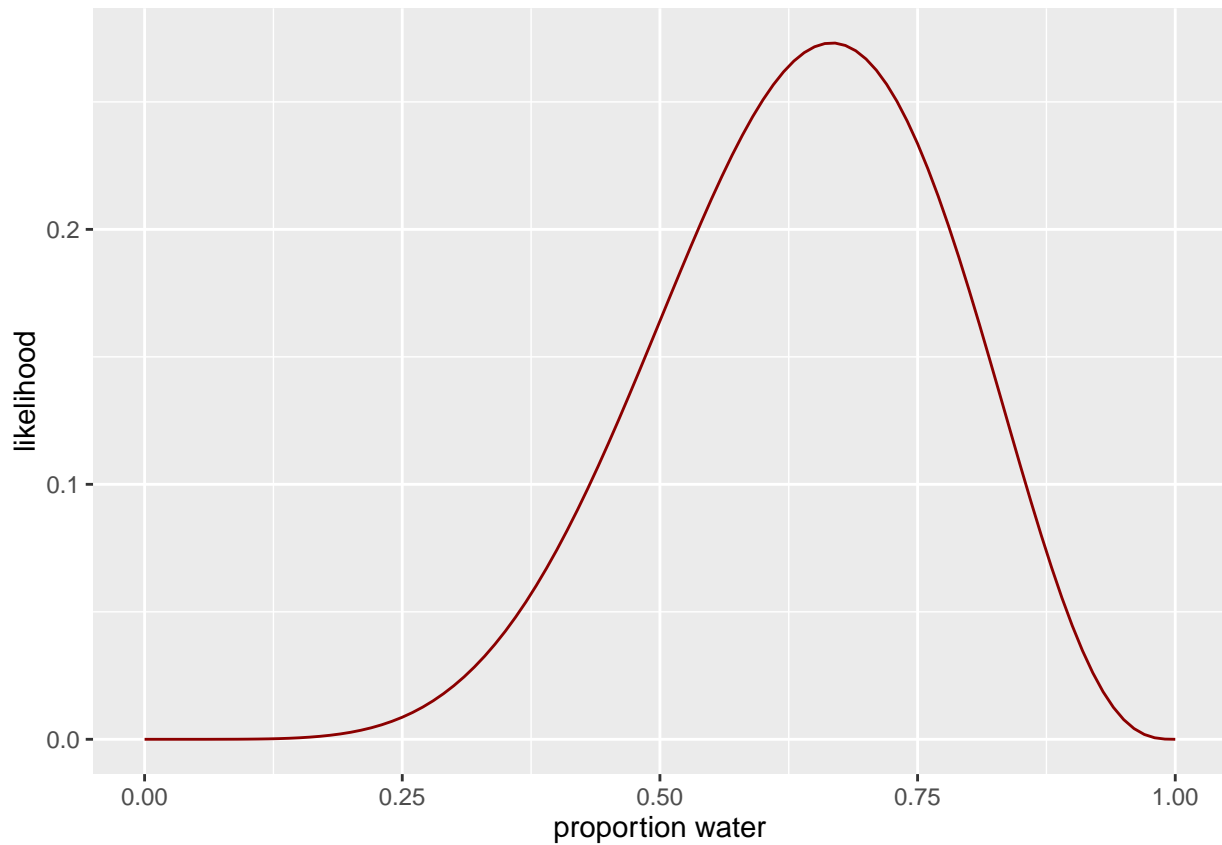
model bayesian inference

display with ggplot2

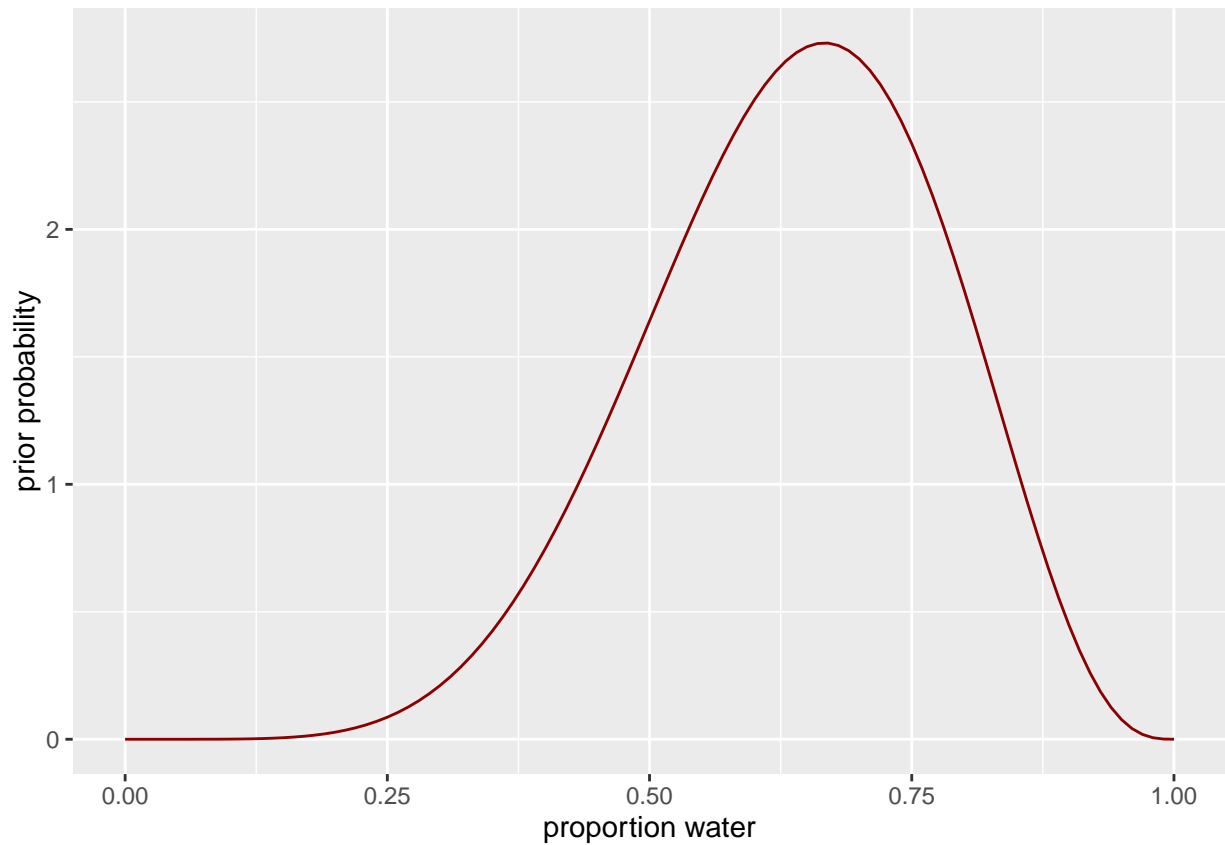
```
data <- data.frame(x = c(0, 1), y = c(0, 2))
plot.prior <- ggplot(data, aes(x, y)) +
  stat_function(fun = dbeta, args = list(shape1 = 1, shape2 = 1),
    color = "darkred") +
  labs(x = "proportion water" , y = "prior probability")
plot.prior
```



```
w <- 6
n <- 9
data <- data.frame(x = c(0, 1), y = c(0, 1))
plot.likelihood <- ggplot(data, aes(x, y)) +
  stat_function(fun = dbinom, args = list(x = w, size = n),
               color = "darkred") +
  labs(x = "proportion water" , y = "likelihood")
plot.likelihood
```

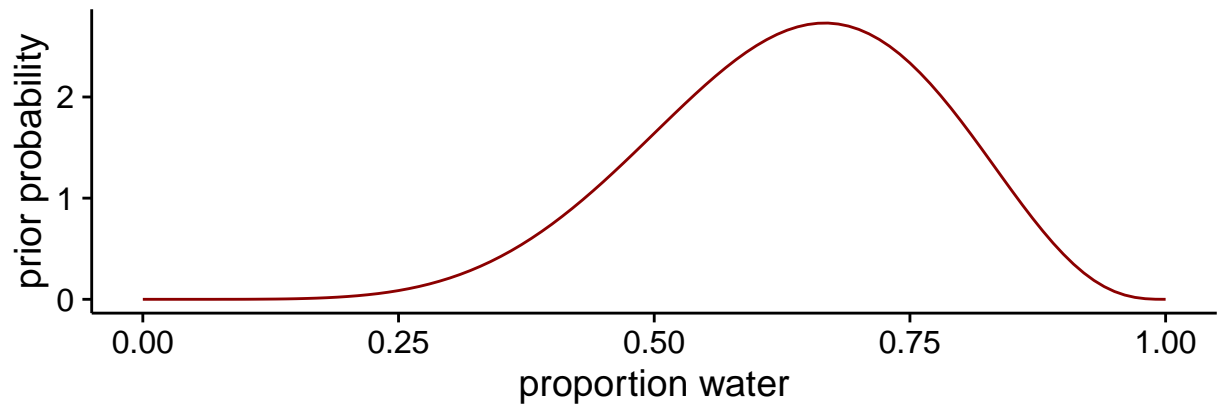
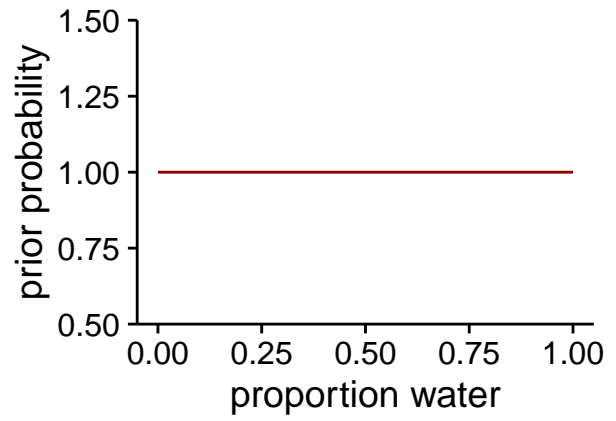
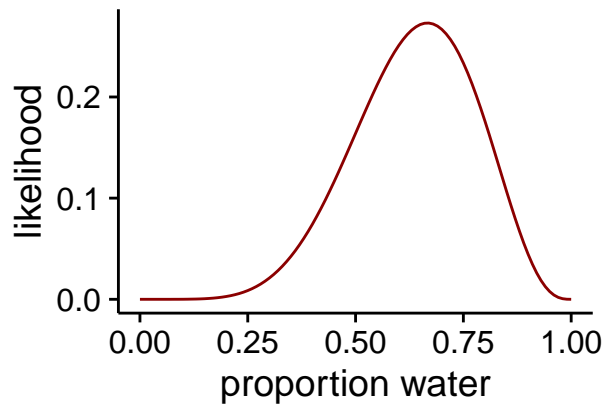


```
data <- data.frame(x = c(0, 1), y = c(0, 2))
plot.posterior <- ggplot(data, aes(x, y)) +
  stat_function(fun = dbeta, args = list(shape1 = w+1, shape2 = n-w+1),
              color = "darkred") +
  labs(x = "proportion water" , y = "prior probability")
plot.posterior
```

Plot prior, likelihood, and posterior.

```
library(cowplot)
quadratic.approximation <- ggdraw() +
  draw_plot(plot.likelihood, 0, .5, .5, .5) +
  draw_plot(plot.prior, .5, .5, .5, .5) +
  draw_plot(plot.posterior, 0, 0, 1, .5)
quadratic.approximation
```



```
library(cowplot)
quadratic.approximation <- ggdraw() +
  draw_plot(plot.likelihood, 0, .5, .5, .5) +
  draw_plot(plot.prior, .5, .5, .5, .5) +
  draw_plot(plot.posterior.approx, 0, 0, 1, .5)
quadratic.approximation
```

