Section 7.3: The Class NP


**Basic Concepts**

Define the following concepts:

(a) \( P \)

(b) \( NP \)

**The Class NP**

*(CLR 36.2-1)* Consider the language

\[
\text{GRAPH} - \text{ISOMORPHISM} = \{ \langle G_1, G_2 \rangle : G_1 \text{ and } G_2 \text{ are isomorphic graphs} \}. 
\]

Prove that \( \text{GRAPH} - \text{ISOMORPHISM} \in \text{NP} \) by describing a polynomial-time algorithm to verify the language.

*(CLR 36.1-4)* Show that the class \( \text{NP} \) of languages is closed under union, intersection, concatenation, and Kleene star. Discuss the closure of \( \text{NP} \) under complement.

*(CLR 36.1-5)* Show that any language in \( \text{NP} \) can be decided by an algorithm running in time \( 2^{O(n^k)} \) for some constant \( k \).

*(CLR 36.1-8)* Let \( \phi \) be a boolean formula constructed from the boolean input variables \( x_1, x_2, \ldots, x_k \), negations \( (\neg) \), AND's \( (\land) \), OR's \( (\lor) \), and parentheses. The formula \( \phi \) is a *tautology* if it evaluates to 1 for every assignment of 1 and 0 to the input variables. Define \( \text{TAUTOLOGY} \) as the language of boolean formulas that are tautologies. Show that \( \text{TAUTOLOGY} \in \text{coNP} \).

*(CLR 36.1-9)* Prove that \( P \subseteq \text{coNP} \).

*(CLR 36.1-10)* Prove that if \( \text{NP} \neq \text{coNP} \), then \( P \neq \text{NP} \).