Section 7.3: Cliques


**Basic Concepts**

Define each of the following concepts:

(a) $P$

(b) $NP$

(c) Undirected graph, $G = (V, E)$

Describe two distinct ways of encoding undirected graphs so that they can be processed by computers.

**Triangles**

Let

$$TRIANGLE = \{\langle G \rangle : G \text{ is an undirected graph containing a triangle}\}$$

Construct a Turing machine which decides $TRIANGLE$.

Show that $TRIANGLE \in P$.

Hint: Calculate the runtime of the Turing machine just constructed.

On input $\langle G \rangle$ you should find a time complexity containing a factor of the form $O\left( \binom{3}{2} \left( \binom{|V|}{3} \right) \right)$. 
k-Cliques

For each positive integer $k$, let

$$CLIQUE(k) = \{ \langle G, k \rangle : G \text{ is an undirected graph containing a } k\text{-clique} \}$$

Construct a Turing machine which decides $CLIQUE(k)$.

Show that $CLIQUE(k) \in P$.

Hint: Calculate the runtime of the Turing machine just constructed.
On input $\langle G, k \rangle$ you should find a time complexity containing a factor of the form $O \left( \binom{k}{2} \binom{|V|}{k} \right)$.

Cliques

Let

$$CLIQUE = \{ \langle G, k \rangle : G \text{ is an undirected graph containing a } k\text{-clique} \}$$

Construct a Turing machine which decides $CLIQUE$.

Show that $CLIQUE \in NP$.

Hint: Calculate the runtime of the Turing machine just constructed.
On input $\langle G, k \rangle$, you should find a time complexity containing a factor of the form $O \left( \binom{k}{2} \binom{|V|}{k} \right)$. If $k$ is fixed, this gives polynomial time, but if $k$ is allowed to grow with $n$, then the runtime becomes exponential. Show, for instance, that $\binom{n/2}{2} \binom{n}{n/2}$ is larger than $2^n$. Stirling’s Formula might be helpful here.